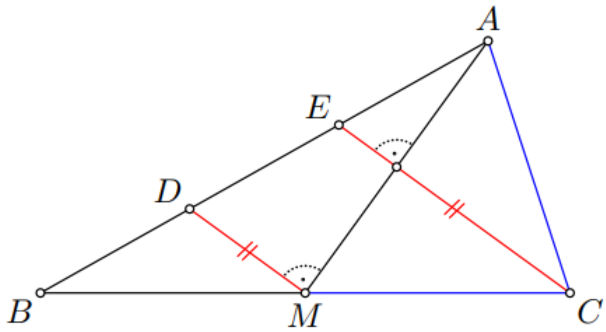
Solution 1. Denote E the midpoint of AD. Then $|BD| = |DE| = |AE|$. The segment DM is the midsegment of triangle BCE, thus $DM \parallel EC$. Since E is a midpoint of AD, line EC pass trough the midpoint of AM. From $|BC| = 2|AC|$ it follows that the triangle CAM is isosceles, thus $CE \perp AM$. Together with $EC \parallel DM$, it means that $AM \perp MD$.



Solution 2. Since n has $d(n)$ divisors, and $d(n) - 2$ of them are smaller than $d(n)$. It follows that all but one numbers from $1, 2, \ldots, d(n)$ divides n.

Suppose that $d(n) > 3$. Between numbers $1, 2, \ldots, d(n)$ are at least two even numbers, so n has an even divisor and $2 \mid n$. The second largest divisor of an even number n is $n/2$, which means $2d(n) = n$.

Suppose $d(n) - 1 \mid n = 2d(n)$. Since $d(n) - 1 \mid 2d(n) - 2$ we have $d(n) - 1 \mid 2$ which is a contradiction with $d(n) > 3$.

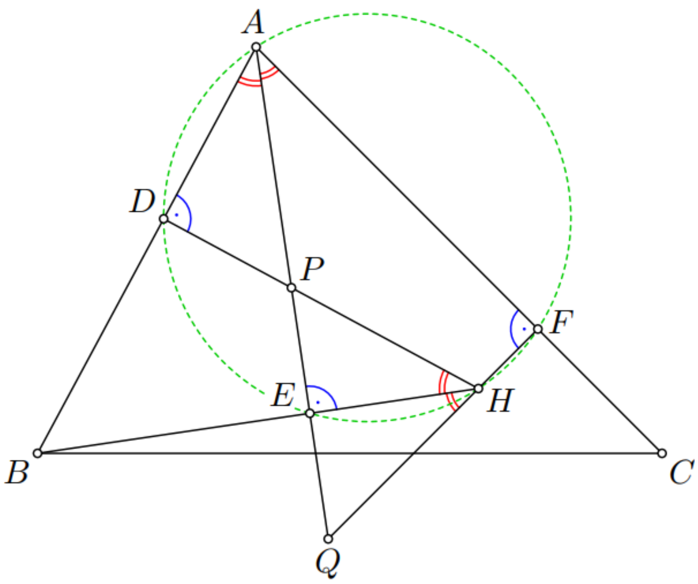
Thus, $d(n) - 1$ is not a divisor of n, which means that all other numbers from $1, 2, \ldots, n$ are divisors of n, in particular $d(n) - 2 \mid 2d(n)$. From this it follows $d(n) - 2 \mid 4$. Since $d(n) > 3$, the only options are $d(n) = 4$ and $d(n) = 6$. This gives us $n = 8$ or $n = 12$. It is easy to check that they both are solutions.

It remains to check values $d(n) \leq 3$:

- $d(n) = 1$. This is not possible since n must have at least two divisors.
- $d(n) = 2$. This is also not possible because, then $d(n) = 2$ should be the smallest divisor of n.
- $d(n) = 3$. This means that n is the square of the prime. Since $3 \mid n$ we have the only option $n = 9$. It is easy to check that this also works.

There are three possible values of n, namely $n = 8$, $n = 9$ and $n = 12$.

Solution 3. Denote D the intersection of HP and AB, E the intersection of BH and AQ, F the intersection of QH and AC. Angles ADH and AEH are right angles since H is the orthocenter of ABP. Frotm the definition of Q we also have $|\angle AFH| = 90^circ$. Thus the points A, D, E, H, F lie on the circle with diameter AH. Since AP is the angle bisector of DAF, we have $|\angle DAP| = |\angle PAF|$, from which it follows $|\angle DHB| = |\angle BHQ|$. Since $PQ \perp BH$, this already means that Q is the reflection of the point P by line BH.



Another solution: Right-anled triangles APD and HPE have equal angles at P and at vertices D and E, to they are similar and have equal angles at A and H. Right-anled triangles AFQ and HEQ have equal angles at Q and at vertices F and E, to they are similar and have equal angles at A and H. This means that HEB is the axes of symmery of PQ.

Solution 4. Suppose that a row has exactly i red cells — then there exist exactly i columns with i red cells (precisely those intersecting the row at red cells). Note that $i \neq 0$ (if $i = 0$, we would have an entirely red column — contradiction). Considering any of these i columns we see (by analogous argument) that there are exactly i rows with i red cells. Moreover, all these rows (and all the columns) are colored in precisely the same way (i.e. have red cells on the intersections with the same set of columns).

This means that there exist positive numbers $i_1, i_2, \ldots, i_k$ so that $i_1+i_2+\ldots+i_k = n$ and the board has precisely i_j columns/rows with exactly i_j red cells (for each $j = 1, 2, \ldots, k$). This means that the total number of red cells, equal to $i_1^2 + i_2^2 + \ldots + i_k^2$ has the same parity as n (i_j and i_j^2 have the same parity), and in consequence — the same parity as n^2. Therefore the total number of blue cells is even.

Solution 5. We will use the following observation: if after some move the result is in $[n-1, n)$ then after at most n following moves the result will exceed n. Indeed, as long as the result is smaller than n, it will increase by at least $\frac{1}{n}$, so if it did not exceed n within $n-1$ moves, it will after the n-th move since the total increase after n moves will be greater than one.

Applying the observation repeatedly, we get that after at most $1+3$ moves the result is greater than 3, after at most $1+3+4$ moves the result is greater than 4 etc. after at most

$$1 + 3 + 4 + \ldots + 24 = 298$$

moves the result is greater than 24. Thus $x > 24$.

To find an upper bound for x we use a similar observation: If after some move the result is smaller than n, then after n moves, the result will be smaller than $n+1$. Indeed, the function $t + \frac{1}{t}$ is increasing for $t \geq 1$. Thus, at the point when the result at first exceeds n, it is at most $n + \frac{1}{n}$. After that, it increase by at most $\frac{1}{n}$ at every move, thus after $n-1$ moves it is still smaller than $n+1$

We know that after 3 moves, the result is smaller than 3. Using the observation repeatedly, we get that after $3+3$ moves the result is smaller than 4, after $3+3+4$ moves the result is smaller than 5 etc. After at most

$$3+3+4+5+\cdots+24=300$$

moves the result is smaller than 25.

Therefore $24<x<25$ which means $\lfloor x\rfloor=24$.