Solution 1. Take $s = S(n)$. Note that $n + s$ and $n - s$ differ by $2s$, so they have the same parity. This means that if $n - s = k^2$, then $n + s \geq (k + 2)^2$, so in particular $2s = (n + s) - (n - s) \geq 4k + 4$, i.e. $s \geq 2(k + 1)$.

Now if $100 \leq n - s < 400$, then $s \leq 21$ and $k \geq 10$, which contradicts the inequality $s \geq 2(k + 1)$. If $400 \leq n - s < 10000$, then $s \leq 36$ and $k \geq 20$, which again gives a contradiction

In general if for some $m \geq 2$ we have $100^m \leq n - s < 100^{m+1}$ (i.e. $n - s$ has $2m + 1$ or $2m+2$ decimal digits), then $k \geq 10^m$ and $s \leq 18(m+1)$, which combined with $s \geq 2(k+1)$ gives $9(m + 1) \geq 10^m + 1$. This is a contradiction since for $m \geq 2$ we have

$$10^m + 1 > \underbrace{999\dots9}_{m} = 9 \cdot \underbrace{111\dots1}_{m} > 9(m + 1).$$

If $n - s$ has at most 2 digits, then (as it is divisible by 9), it could only be 9, 36, or 81. Direct verification shows that only for $(n, s) = (17, 8)$ we get a valid solution.

Solution 2. Denote the numbers on the board by $a_1, a_2, \dots, a_{2023}$. By writing the string $(a_i, a_{i+1}, \dots, a_{i+2k})$ in opposite order, the number a_j, for $j \leq k + i$, exchanges with the number $a_{2k+2i-j}$. Indices j and $2k + 2i - j$ are either both even or both odd. Thus we can never change the number from even position with the number from odd position.

If we pick numbers a_i, a_{i+1} and a_{i+2} and write them in opposite order we have exchanged only numbers a_i and a_{i+2}. Clearly by making such operations we can achieve any ordering of numbers on even positions and also any ordering of the numbers on odd positions.

Therefore, we can reach $1011! \cdot 1012!$ possible orderings.

Solution 3. Let $N(A)$ be the only person not liked by guest A. Arrange all guests in a line $(A_1, A_2, \dots, A_n)$ in the following way. Choose A_1 arbitrarily and put $A_2 = N(A_1)$. For $i \geq 2$ if $N(A_i) = A_j$ for some $j < i$, then choose A_{i+1} arbitrarily from the remaining guests. Otherwise take $A_{i+1} = N(A_i)$.

Now let the guests enter the room one by one respecting the order of the line. A_1 chooses any table, and then every A_{i+1} sits by a table with no A_i and no $N(A_{i+1})$ (there is always at least one such table). After everyone is seated, the conditions are clearly met.

Solution 4. Let D be the foot of the perpendicular from A to BC. We will prove that D is the point of intersection of lines p and q. First, we show that D lies on p. By easy angle chasing we get that:

$$|\sphericalangle MBP| = |\sphericalangle PBC| = |\sphericalangle BPM|,$$

therefore $|MB| = |MP|$. We know that $|\sphericalangle ADB| = 90^\circ$, which means that D lies on circle centered at M with radius $\frac{|AB|}{2} = |MB|$, so $|MD| = |MB| = |MP|$. Therefore triangle MDP is isosceles, $|\sphericalangle MPD| = |\sphericalangle MDP|$ and using the fact that points A and D are symmetric with respect to line MN, we get that

$$|\sphericalangle MPD| = |\sphericalangle MDP| = |\sphericalangle MAP|,$$

which means, that PD is tangent to circle AMP at point P. Thus, point D lies on p. Analogously, we get that D lies on q.

Solution 5. A key observation is that every operation decreases the sum of the numbers by 1. Negative numbers will never appear. Consequently, after $x + y + z$ operations we end up with the triple 0, 0, 0 and we are done.

(a) The maximum possible number of operations will be attained for triples with maximum sum. Consider a triple x, y, z such that $x \geq y \geq z$ and $xy + yz + zx = 1000$. If $y = z = 1$, there is a contradiction $2x + 1 = 1000$. If $y = 2$ and $z = 1$, then $3x + 2 = 1000$, again a contradiction. If $y = 2$ a $z = 2$, we have $x = 249$ and we can perform 253 operations. If $y = 3$ and $z = 1$, then $4x + 3 = 1000$, again a contradiction. If $y = 3$ and $z = 2$, we have a contradiction $5x + 6 = 1000$. In every other case $y + z \geq 6$, hence $x < 1000/(y + z) < 167$. Since $1000 > xy \geq y^2$, both y and z are at most 31, thus in every other case $x + y + z < 167 + 2 \cdot 31 < 253$. Thus the triple $(2, 2, 249)$ allows the maximum number of operations.

(b) The minimum possible number of operations will be attained for triples with minimum sum. Since $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx) \geq 3(xy + yz + zx) = 3000$, we have $x + y + z \geq 55$. The triple $(20, 20, 15)$ have the sum equal to 55 and sastisfy $(*)$, thus it is a triple that allows minimal number of operations.

Remark: Rather than guess the final triple in part (b) one can look at the remainders modulo 5 and figure out that x, y, z are all divisible by 5. After that, it remains to check only a few cases and one can show that this triple is, in fact, unique.

Solution 6. Answer: $\sqrt{5}$.

Let $AB = a$, $AD = b$. By the problem's conditions we get $BE = 2/a$, $DF = 2/b$ and in consequence

$$2 = 2[ECF] = CE \cdot CF = (b - 2/a)(a - 2/b) = ab - 4 + 4/(ab).$$

Taking $x = ab - 3$ to be the desired area, we obtain

$$(3 - x) = 4/(x + 3), \quad \text{so} \quad (3 - x)(3 + x) = 4,$$

i.e. $x^2 = 5$.

Remark. After reaching the algebraic form, we can of course just algorithmically solve a quadratic equation in ab. The funny thing is that the natural auxiliary variable x turns out to be the one giving the canonical form.